SOME ASPECTS OF BOUNDARY LAYER TRANSITION AT SUBSONIC SPEEDS

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Summary—The paper presents the results of experimental investigations carried out by a small research group in Tokyo concerning the instability leading to transition in three types of laminar flow: the wake behind a flat plate, the boundary layer along a flat plate with artificially introduced streamwise vortices, and the boundary layer along a concave plate. Attempt is made to obtain a unified conceptual picture of the development of laminar instability, with special interest in the nonlinear and three-dimensional phenomena.

INTRODUCTION

IT is now generally accepted that the transition from laminar to turbulent flow in the boundary layer is preceded by the appearance of fluctuations of the type predicted by the linearized theory of laminar instability provided that all sources of disturbance are sufficiently small. This belief is based primarily on the extensive confirmation by Schubauer and Skramstad[6] for the flow along a flat plate, but there is little doubt that the initiation of disturbances leading to transition in other types of flow may also be identified with those produced by laminar instability. Then the questions arise as to how the fluctuations originate and how they grow until turbulence ensues. The first question, however, contains an inherent difficulty in the sense that it is impossible to eliminate the irregularities in flow and the disturbances of external origin. Even if the disturbances could be reduced to such an extent that they were undetectable by the usual instruments, it would appear still possible for them to affect the transition to a marked degree.

It is rather more advantageous to minimize the effect of inherent residual disturbances by the controlled introduction of artificial disturbances. This has already been done by Schubauer and Skramstad[6] by causing a fine ribbon to vibrate in the boundary layer to produce a two-dimensional travelling wave, which could be identified with the disturbance assumed in the linearized stability theory of Tollmien[1] and Schlichting[3,4], and now generally called a Tollmien–Schlichting wave.

The hot-wire observation indicates that the wave, when initially weak, develops downstream in the way predicted by theory. When the wave is initially not so weak, however, its development begins to deviate from that predicted by theory at some location downstream of the ribbon, certain nonlinear effect manifesting itself before the eventual breakdown of laminar flow. The most striking feature of this nonlinearity is the wave development into a three-dimensional configuration, even though the initial disturbance is purely two-dimensional. Through the investigations of Fales[10], Weske[11], Hama, Long Hegarty[12], Schubauer[13], Klebanoff and Tidstrom[15], it is now known that the formation of three-dimensional disturbances is a prerequisite to turbulent flow in the boundary layer along a flat plate. Associated with this three-dimensionality there is evidence of the existence of vortices with axes in streamwise direction.

In view of our still insufficient understanding of the complicated development of disturbances, it would be of some value to put on record the results of the experimental investigations undertaken by our research group concerning the instability leading to transition in three types of laminar flow: the wake behind a flat plate, the boundary layer along a flat plate with artificially introduced streamwise vortices, and the boundary layer along a concave plate. To make the situation as simple as possible the affect of streamwise pressure gradient and compressibility were eliminated. Artificial disturbances of controlled characteristics were employed to suppress the undesirable effect of residual disturbances. The investigation in the first case was expected to give informations as to the difference, if any, in the nonlinear development of disturbances between flows with and without solid boundaries. In the second case, the investigation was made on the development of disturbance under the influence of spanwise variation in boundary layer thickness, which was produced by the artificial vortices. In the third case, the streamwise vortices produced by centrifugal force served a similar purpose. It is hoped that the results of these investigations will throw some light on the mechanism of transition.

The investigations were carried out by our small informal group for boundary layer and turbulence research, with the financial assistance from the Ministry of Education of Japan. The members of the group are H. Komoda of Nihon University, J. Sakagami and M. Mochizuki of Ochanomizu University, T. Matsui of Gifu University, and H. Sato, M. Iuchi, Y. Aihara, K. Kuriki and myself at the University of Tokyo. Detailed account of the investigations will be published elsewhere by the individual members.

TRANSITION IN THE WAKE BEHIND A FLAT PLATE

Measurements were made by Sato and Kuriki, University of Tokyo, on the wake downstream of a flat plate mounted in a 60 by 60 cm low-turbulence wind tunnel. The plate was 30 cm long, 60 cm wide, and 3 mm thick with sharpened leading and trailing edges. The results to be presented were obtained at a free-stream velocity of 10 m/sec in which the boundary layer was completely laminar over the plate, having a Reynolds number of 800 based on the displacement thickness at the trailing edge.
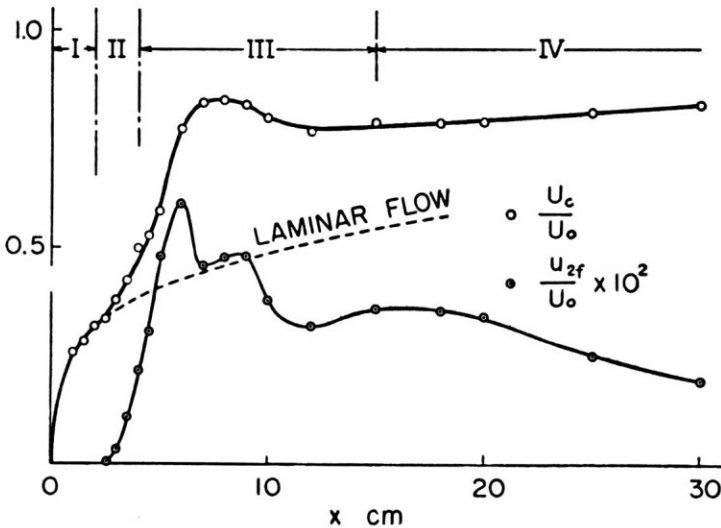

FIG. 1. Downstream development of mean velocity and root-mean-square intensity of velocity fluctuation at twice the fundamental frequency at the wake center. $U = 10$ m/sec.

Figure 1 shows the mean velocity U_c at the wake center in terms of the free-stream velocity U_0 as a function of the distance x measured downstream from the trailing edge. In the region denoted by I, closely behind the trailing edge, the development of U_c is in good agreement with laminar flow theory[2]. In the region II, the development of U_c begins to deviate from theoretical prediction, and the velocity fluctuation, having a definite frequency f, makes its appearance. The characteristics of the fluctuation agree well with those obtained by the linearized stability theory, in which a small disturbance of the form of traveling wave is assumed to be super-imposed on the steady flow. An example of comparison with theory is shown in Fig. 2 of the distributions of phase and amplitude of the fluctuation at the station $x = 4$ cm, y is the distance across the wake measured from its center, b is the half semi-width of the wake, and ϕ_f and u_f are the phase and root-mean-square intensity, respectively, of the streamwise

velocity fluctuation at a frequency $f = 730$ c/s. This frequency is found to be the one for which the spatial amplification rate is a maximum according to stability theory, the circumstance being similar to that previously observed in the case of laminar jet[16]. The phase shift of 180 degrees at $y = 0$ indicates the anti-symmetric behavior of velocity fluctuation.
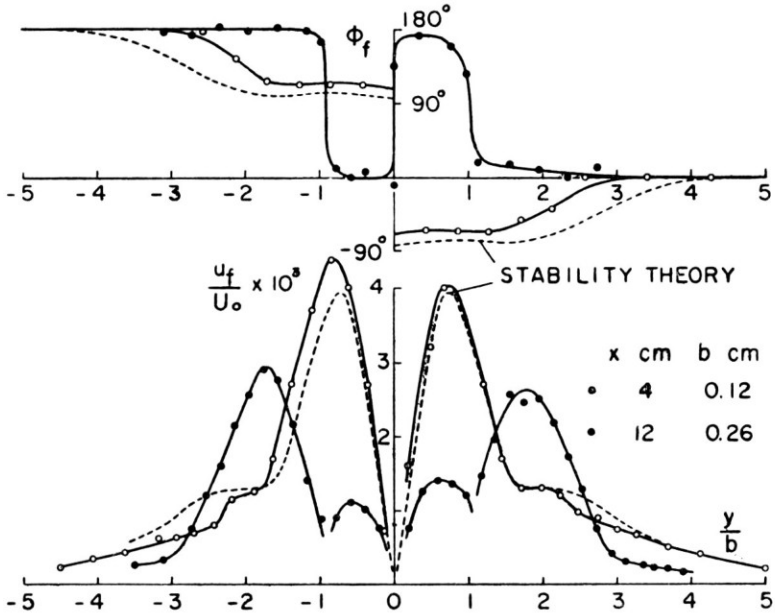

FIG. 2. Distributions of phase and root-mean-square intensity of velocity fluctuation at a frequency of 730 c/sec at the stations $x = 4$ and 12 cm. $U_0 = 10$ m/sec.

Nonlinear effects seem to come into being in the region III in that the velocity fluctuation exhibits some deviations from theoretical prediction. The fluctuation at twice the fundamental frequency is observed at the wake center. In Fig. 1 is included the downstream development of the root-mean-square intensity u_{2f} of fluctuation at a frequency $2f$. More striking feature is seen from Fig. 2, in which the distributions ϕ_f and u_f are presented also for the station $x = 12$ cm. The phase shift of 180 degrees occurs three times here as contrasted with only one shift for the upstream station $x = 4$ cm. The understanding might be facilitated if the disturbance assumed in theory is interpreted as that produced by a single row of alternating vortices with centers aligned at $y = 0$. Then the evolution observed in the region III is understood by speculating the vortices to be rearranged in a staggered double row, namely in the form of Kármán street. This view is supported by Taneda's water-tank observation[14] of a double row of vortices in the wake behind a flat plate.

The velocity fluctuation becomes three-dimensional only after arriving in the region IV. This is judged by the measurement of fluctuation in z-direction, which is parallel to the trailing edge. The z-component of fluctuation increases gradually downstream until a fully developed turbulent wake is established.

The measurements referred above were carried out to better advantage by exciting fluctuations by a loudspeaker installed outside of the tunnel and fed by an oscillator of the same frequency as the natural fluctuation. It is interesting to note that when the exciting frequency was a little lower than the natural frequency, the natural fluctuation was suppressed so that two kinds of fluctuations were observed in the III, one at the natural and the other at the exciting frequencies, respectively.

TRANSITION IN THE BOUNDARY LAYER ALONG A FLAT PLATE WITH ARTIFICIALLY INTRODUCED STREAMWISE VORTICES

Measurements were made by Komoda of Nihon University on the boundary layer along a flat plate mounted in a 20 by 60 cm low-turbulence wind tunnel having an adjustable wall for eliminating pressure gradient. The plate was 300 cm long, 60 cm wide, and 5 cm thick with a sharpened leading edge. The reference axes are taken in such a way that x is measured along the flat plate from the leading edge in the streamwise direction, y in the direction perpendicular to the plate, and z along the plate in the spanwise direction perpendicular to x.

Controlled vortices with axes in x-direction were generated outside the boundary layer by means of five small wings of 3 cm span in a spanwise row at a height $y = 2 \cdot 1$ cm at a location $x = 40 \cdot 7$ cm. The spacing of wings was 6 cm, approximately twice the natural wavelength of mean-velocity variation in spanwise direction. Then the boundary layer was found to exhibit a spanwise periodicity in thickness, which was minimum along the centerline of the wing, but the velocity profiles across the boundary layer were very close to the Blasius profile. Figure 3 shows a back view of the lines of constant mean velocity in the boundary layer in the section $x = 85 \cdot 5$ cm. For later reference the longitudinal sections are denoted by A, B, C, D and E; A for the section of minimum boundary layer thickness, and E for that of maximum thickness. As a measure of the spanwise variation of boundary layer thickness the ratio $T = (\delta_E^* - \delta_A^*)/\delta_A^*$ is used, where δ_A^* and δ_E^* are the displacement thickness in the sections A and E, respectively. T had a value of $0 \cdot 30$ in the case shown in Fig. 3, but could be varied from $0 \cdot 1$ to $0 \cdot 9$ by changing the angle attack of the wings.

Controlled two-dimensional waves were produced at a location $x = 75.5$ cm by Schubauer's technique of vibrating ribbon [6]. An alternating current

of the desired frequency was passed through a thin phosphor-bronze ribbon in the presence of a magnetic field from a permanent magnet. The results to be presented were obtained at a free-stream velocity U_0 of 7 m/sec and an exciting frequency f of 60 c/s to the ribbon. The Reynolds number based on the mean boundary layer displacement thickness was 1100 at the location of the ribbon, so that the wave of the above
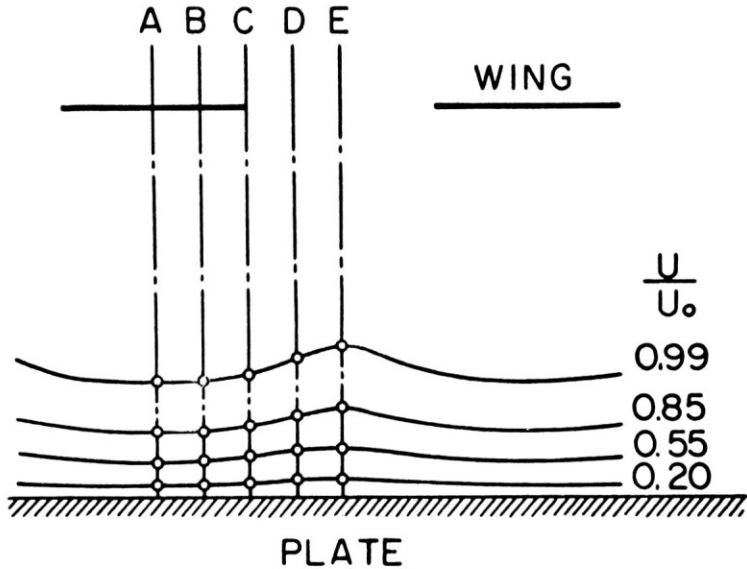

FIG. 3. Back view of wings and lines of constant mean velocity in the boundary layer at the station $x = 85 \cdot 5$ cm. The vertical scale is magnified by a factor of 3.

frequency was the one to be amplified according to the linearized stability theory, which is originally due to Tollmien[1], and later refined by Schlichting[3,4] and Lin[8]. Hot-wire measurements were made of the streamwise velocity fluctuations to trace the downstream development of wave intensity and wave phase.

Figure 4 shows the distributions across the boundary layer of wave phase ϕ_f and root-mean-square wave intensity u_f at a frequency $f = 60$ c/s at various stages of downstream development in the section A. The location of the station is defined by the distance $x - x_r$ measured from the location of vibrating ribbon, $x_r = 75 \cdot 5$ cm. The abscissa is made non-dimensional by dividing the distance y from the plate by the boundary layer thickness δ defined as equal to $3\delta^*$. The results are for the case of weak exciting current to the ribbon and of thickness-variation factor $T = 0 \cdot 30$. Similar results are shown in Fig. 5 and 6 for the sections D and E, respectively. The phase is the value relative to that observed near the outer edge of the boundary layer in the section A.

The distribution of wave intensity in the section A, and also in the sections B and C (not shown in the figure), are essentially of the type as predicted by the Tollmien-Schlichting stability theory, indicating a sign of the familiar phase shift in the outer region. But the intensity distribu-
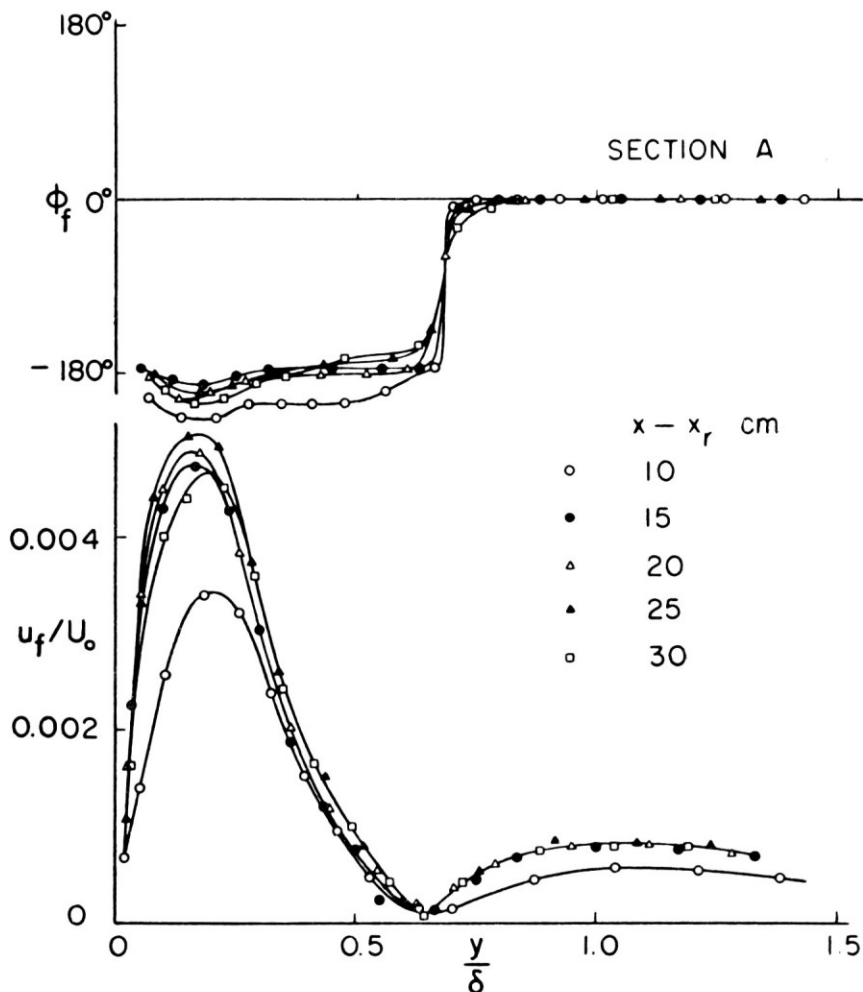
FIG. 4. Distributions across the boundary layer of wave phase and root-mean-square wave intensity at a frequency of 60 c/sec at various distances from the ribbon. Linear development, $T = 0.30$. Section A.

tions in the sections D and E are somewhat different and characterized by the M-shaped curve having a deep indentation at the height of about one fifth of the boundary layer thickness. A second phase shift amounting to nearly 180 degrees occurs at the same height. Referred to the section A there is a phase lag near the outer edge of the boundary layer. If the velocity

fluctuation there is interpreted as that caused by the vorticity wave within
the boundary layer, the lag in phase suggests the lag in wave front for
the sections D and E.

Thus the intensity distributions are different in different longitudinal
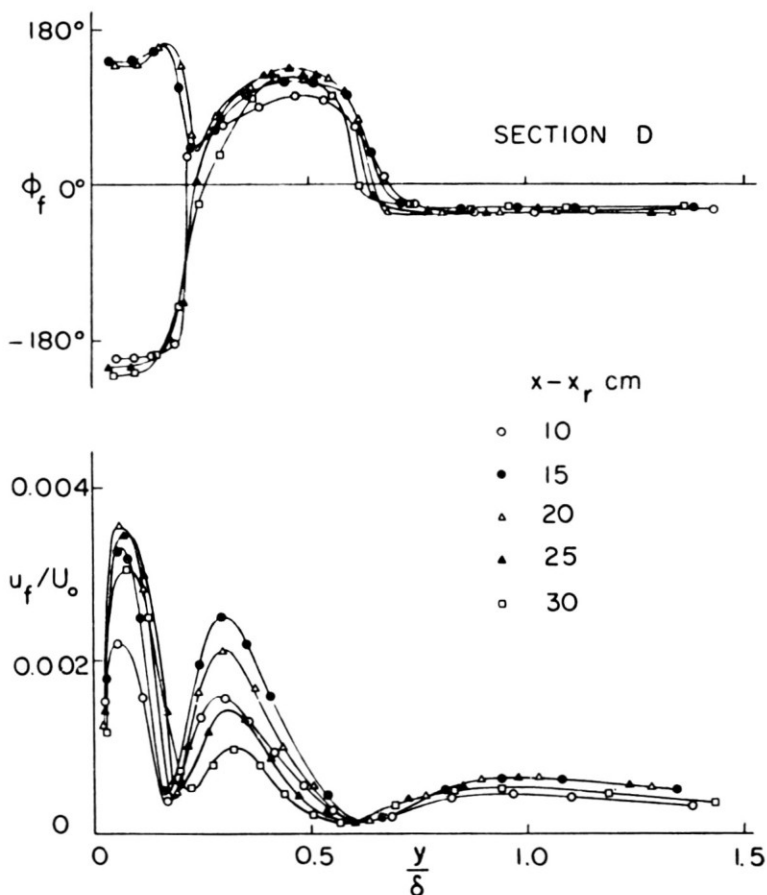sections, but they preserve the shape, or in other words, are in proportion

FIG. 5. Distributions across the boundary layer of wave phase and root-mean-
square wave intensity at a frequency of 60 c/sec at various distances from the
ribbon. Linear development, $T = 0.30$. Section D.

to the initial disturbance so far as it is weak, or the exciting current is
weak. This behavior is called the linear development. The disturbance
is first amplified, but then damped out eventually, irrespective of the
presence of spanwise variation of boundary layer thickness. Increase
in the value of T simply makes the change occur earlier in the intensity
distribution in some of the sections, especially the section E.

When the exciting current is increased beyond a certain limit so that the initial disturbance is no longer weak, the wave development begins to deviate from the linear development. The resulting nonlinearity manifests itself by the rapid amplification of fluctuation, the distortion in oscillograph wave form of fluctuation, and the distortion in mean velocity profile.
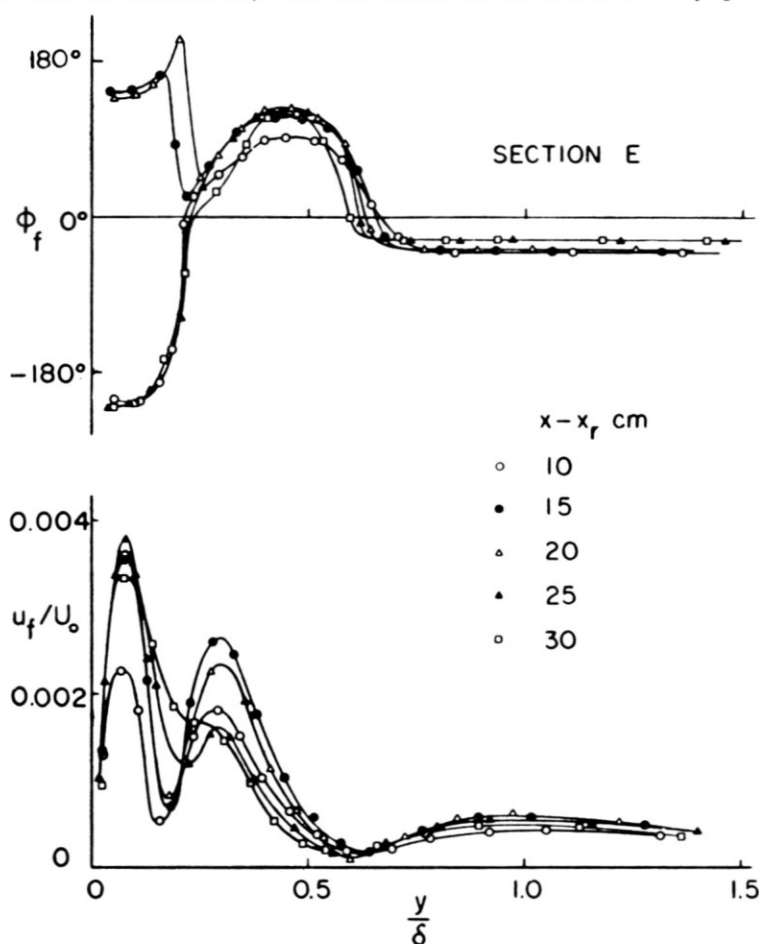

FIG. 6. Distributions across the boundary layer of wave phase and root-mean-square wave intensity at a frequency of 60 c/sec at various distances from the ribbon. Linear development, $T = 0.30$. Section E.

It is catastrophic in that its occurrence inevitably leads to the breakdown of laminar flow, and hence the onset of turbulence. This nonlinear development takes place progressively farther upstream as the initial disturbance is increased or the thickness–variation factor T is increased.

Figure 7 shows the distributions of ϕ_f and u_f at the station $x - x_r = 25$ cm in the sections A, D and E for the case of strong excitation and $T=0.30$.

The intensity distributions at the stations $x-x_r = 10$ and 15 cm being similar to those shown in Fig. 4, 5 and 6 for weak excitation, except that they are about four times larger, the amplification of intensity between the stations $x-x_r = 15$ and 25 cm is quite considerable. In the section A, the amplification takes place mainly within the layer between the peak
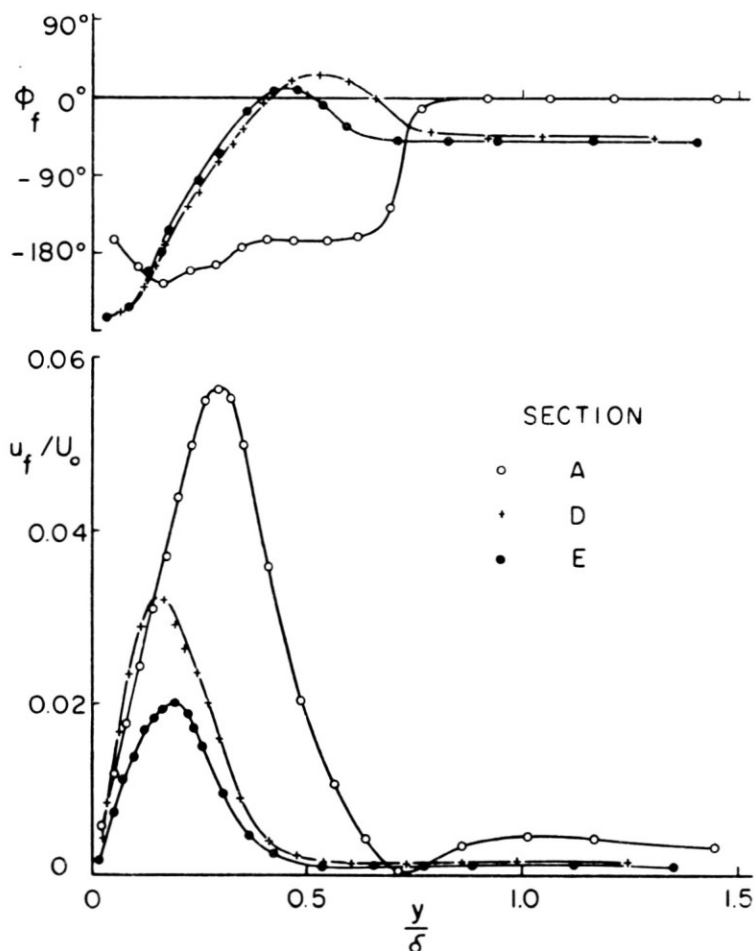

FIG. 7. Distributions across the boundary layer of wave phase and root-mean-square wave intensity at a frequency of 60 c/sec at the station $x - x_r = 25$ cm in the sections A, D and E. Nonlinear development, $T = 0.30$.

intensity and the outer phase shift, thus resulting the center of gravity of intensity distribution to move outward. In the sections D and E, the M-shaped distribution of intensity is distorted into a single-peak distribution of large amplitude. The phase distribution becomes more gradual, but the phase lag near the edge of the boundary layer increases downstream.

In the course of the nonlinear wave development, the wavelength increases gradually downstream in contrast to the linear development in which it is constant. Increase in wavelength is greater in the section A as compared with that in the section E, and this is accounted for by the above mentioned outward shift of the center of gravity of intensity dis-
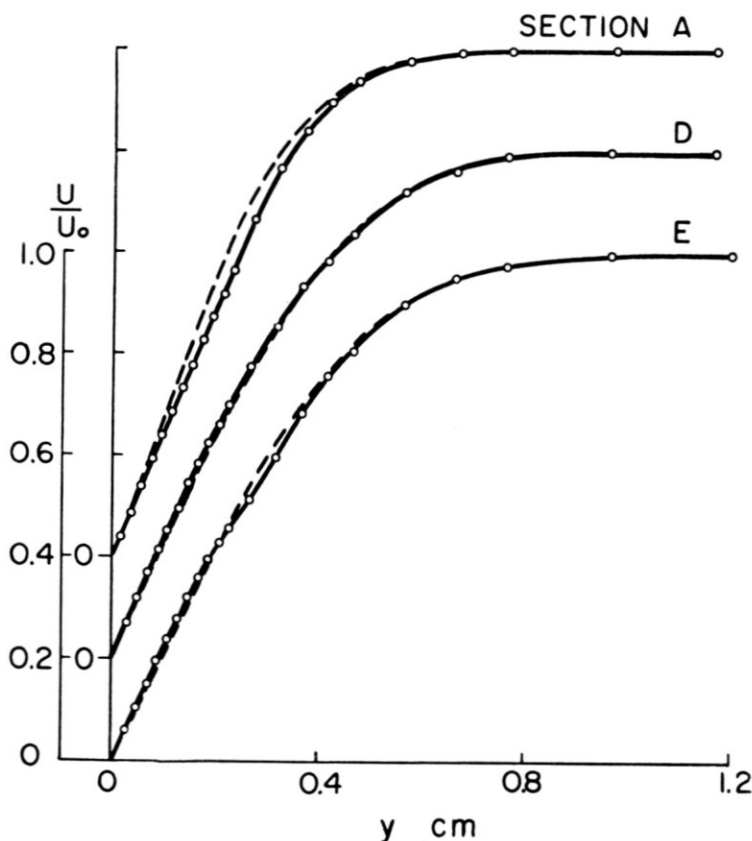

FIG. 8. Mean velocity profiles at the station $x - x_r = 25$ cm in the sections A, D and E. Nonlinear development. The dotted line shows the profile for the linear development in the same section.

tribution. The increase in wavelength, together with the increase in phase lag near the edge boundary layer in the section E, suggests a distortion to proceed downstream in the wave front in such a way that the section A advances in phase relative to the section E.

So far as the wave development is linear, the mean velocity profile does not change; it maintains the Blasius profile within the accuracy of the measurements. In the nonlinear development, however, some distortion takes place in mean velocity profile. As compared with the Blasius

profile, it is fuller in the section D, but thinner in the sections A and E. Figure 8 shows the mean velocity profiles at the station $x-x_r = 25$ cm in three sections, A, D and E. The dotted line shows the profile obtained for weak excitation; it is the Blasius profile.

First appearance of breakdown of laminar flow seems to depend on the thickness-variation factor T. If T is small, the amplification in the section A is catastrophic in that the first breakdown appears near the
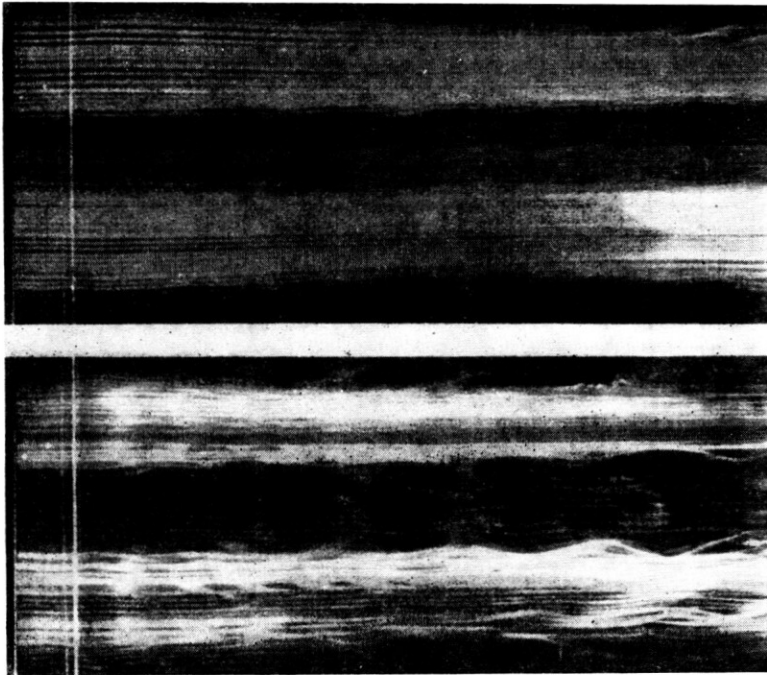

FIG. 9. Smoke photographs showing the flow pattern in the boundary layer on a flat plate. Upper for linear development (weak disturbance); lower for nonlinear development (strong disturbance). Free-stream direction is from left to right.

phase shift of this section. This is in agreement with the observation of Klebanoff and Tidstrom [13, 15] on the natural transition on a flat plate. If T is large, however, the amplification in the section D is so rapid that the breakdown occurs between the intensity peak and the plate surface. The demarcating value of T seems to lie between 0·3 and 0·6. It should be mentioned, however, that the breakdown is the consequence of three-dimensional development of the wave as a whole. There is certainly present a mechanism by which the energy is transferred from one longitudinal section to another so that the breakdown cannot occur as a localized process.

Visual observations were made in air by Sakagami and Mochizuki of Ochanomizu University, and in water by Matsui of Gifu University, the experimental arrangement being similar to Komoda's measurements. Figure 9 shows the photographs taken by Sakagami and Mochizuki by the aid of smoke produced by liquid paraffin sprayed on a fine ribbon located a little downstream of the vibrating ribbon. The exciting current is weak in the upper picture, which indicates a constant wavelength. The
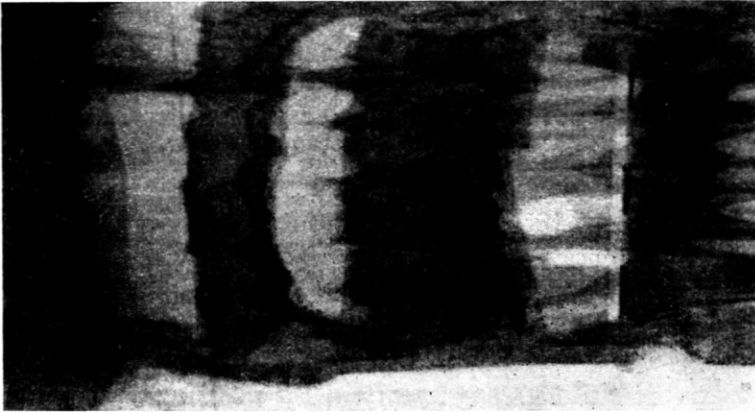

FIG. 10. Dye photograph showing the flow pattern in the boundary layer on a flat plate. Nonlinear development (strong disturbance). Free-stream direction is from left to right.

current is strong in the lower picture, which shows the downstream increase in wavelength as well as the remarkable distortion in wave front. Figure 10 shows the photograph taken by Matsui by the aid of colored film produced by methylene blue dye sprayed on a fine ribbon located a little upstream of the vibrating ribbon. The exciting current is strong enough to produce a nonlinear development. As exhibited by the concentrated dye streak with its spanwise distortion increasing downstream, the wave appears to be strongly amplified to form a vortex filament, which is deformed into a wavy configuration in downstream development. The result reconfirms the previous observation by Hama, Long and Hegarty[12] on the transition induced by a roughness element and the recent observation by Hama[17] on the transition induced by a vibrating ribbon. Hama's statement that the downstream top of the distorted vortex filament travels faster and is lifted outward appears to be in agreement with the above mentioned outward shift of the center of gravity of intensity distribution in the section A and the increase in phase lag near the edge of the boundary layer in the section E.

TRANSITION IN THE BOUNDARY LAYER ALONG A CONCAVE PLATE

Measurements were made by Iuchi and Aihara, University of Tokyo, on the boundary layer along a curved plate mounted in a 100 by 100 cm wind tunnel. The plate was 260 cm long and 100 cm wide; it was flat over the forward portion of 40 cm length, concave with 500 cm radius of curvature over the middle portion of 150 cm length, and convex over the rear portion. The pressure gradient was eliminated by adjusting the opposite auxiliary wall over the forward and middle portions, the latter of which was used for the measurements. The reference axes are taken in such way that x is measured along the plate from the leading edge in the streamwise direction, y in the direction perpendicular to the plate, and z in the spanwise direction perpendicular to x and y. Measurements were made at the free-stream velocity U_0 (velocity outside the boundary layer) of 3 to 12 m/sec.

There are a number of indirect evidences indicating that a laminar flow along a concave plate cannot remain two-dimensional. Instead, it rolls up into a system of alternating vortices with axes in the streamwise direction. The pertinent theoretical analysis was first carried out by Görtler[5] by assuming a small disturbance of the form of streamwise vortices superimposed on the basic boundary layer flow. The analysis yielded a parameter that served as a measure of stability of flow, $R(\delta^*/r)^{1/2}$, where r is the radius of curvature of the concave surface, δ^* the displacement thickness of the boundary layer, and R the Reynolds number based on δ^* and U_0. Subsequent experiment by Liepmann[7] verified the important role of the parameter in determining the location of transition. However, no measurement seems to have been carried out since then to ascertain whether the vortices of the type assumed in theory do really exist, except only simple observations of their trace as revealed by the china-clay record on the plate surface. The fluid motion associated with the vortices being almost steady, the fluctuation measurement by hot-wire anemometer is incapable of determining the structure of the vortices.

Figure 11 shows a typical result at $U_0=7$ m/sec of the measurement of mean velocity U obtained by traversing a hot-wire anemometer in z-direction at a fixed height y from the plate. The spanwise variation of U is considered as produced by the system of alternating streamwise vortices superimposed on the Blasius flow along the plate. Therefore, the number of waves β contained within the spanwise distance of 2π cm is determined from the result shown in Fig. 11 as well as other similar results, and compared with the prediction by linearized theory, which is originally due to Görtler[5], but subsequently modified by Smith[9]. The comparison is shown in Fig. 12, where the parameter $R(\delta^*/r)^{1/2}$ is plotted against the

non-dimensional wave number $\beta\delta^*$, and σ is the spatial amplification rate of the superimposed disturbance. The experimental points appear to lie on a curve representing the maximum theoretical amplification, namely, the maximum of the integral of σ with respect to x. It is possible, moreover, to
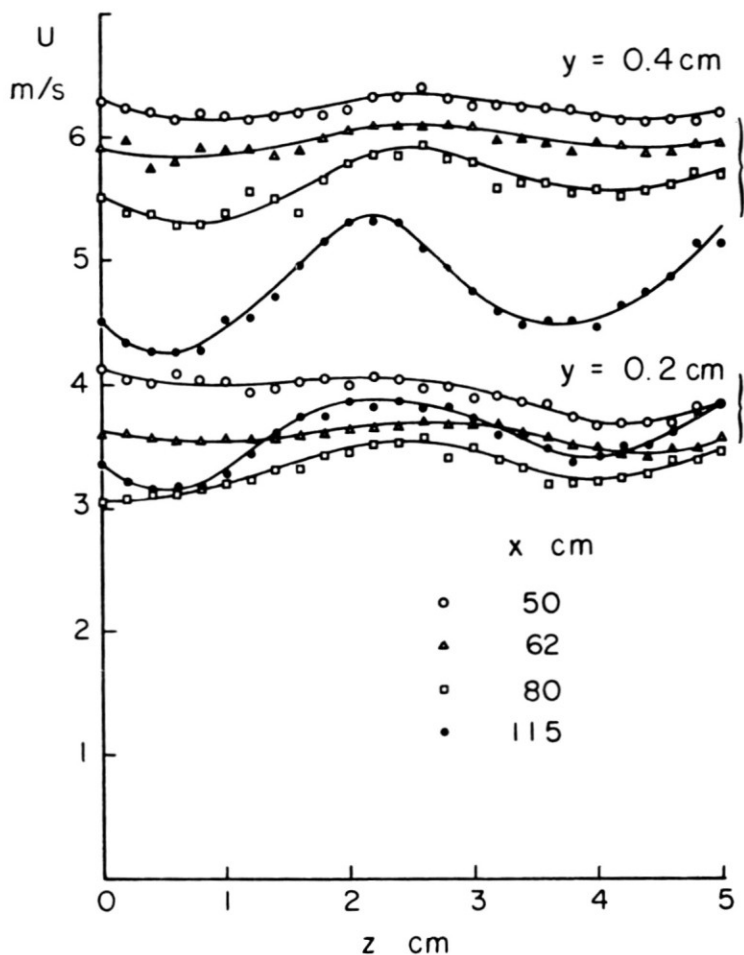

FIG. 11. Distribution of mean velocity in the boundary layer along a curved plate in z direction at two fixed heights y at four distances x from the leading edge. Free-stream velocity $U_0 = 7$ m/sec.

compare the amplitude of the spanwise variation of U with that obtained by theory. The spanwise variation of mean velocity is thus identified with that produced by the system of streamwise vortices.

As seen from Figs. 11 and 12, the vortices are in the unstable range. But the amplification is rather small, even at the downstream station ($x = 115$ cm), where the transition to turbulent flow is detected. It seems

highly probable that the essential effect of vortices on transition is simply to induce a spanwise variation of boundary layer thickness.

Additional measurements were therefore made by producing the amplified Tollmien–Schlichting waves by means of a vibrating ribbon. The downstream development of the waves was traced and found to be not
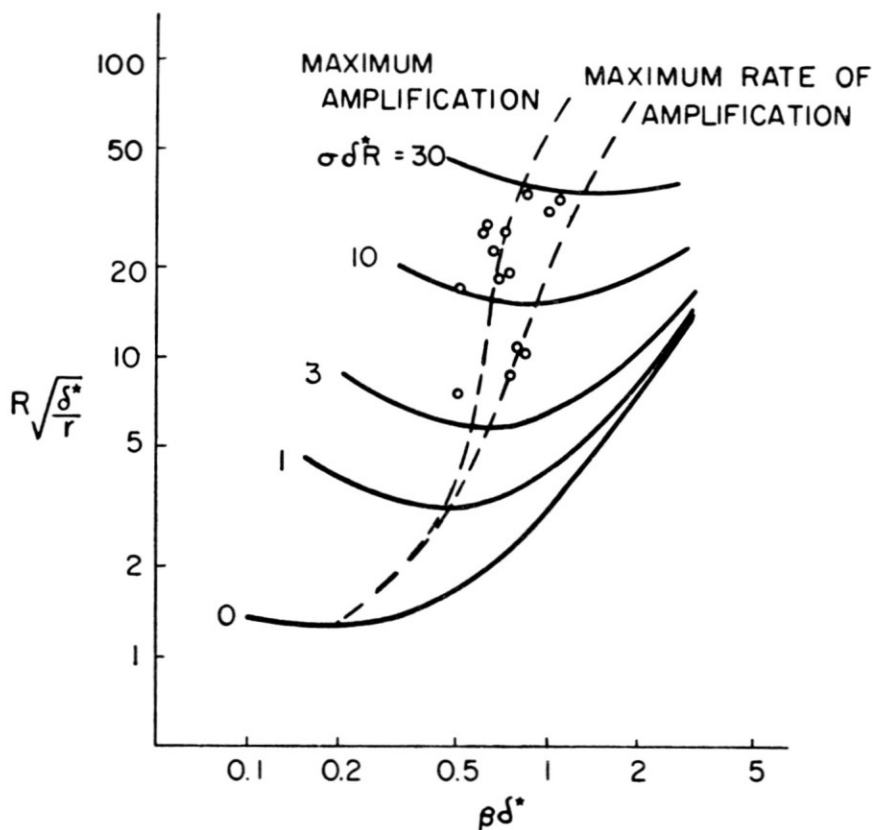

FIG. 12. Comparison of wave number of streamwise vortices observed on the curved plate with theoretical prediction.

much different from, but not exactly the same, as that previously observed in the boundary layer along a flat plate with artificially introduced streamwise vortices. Unfortunately, however, the free-stream turbulence in this experiment was not sufficiently low, so that it was not possible to decide whether the difference came from the effect of free-stream turbulence or from the direct effect of streamwise vortices. It is hoped that the question will be answered in the not too distant future by carrying out the measurements in a low-turbulent stream.

CONCLUSION

The result presented above of our investigations suggest the following conceptual picture of the instability of laminar boundary layer along a flat plate leading to turbulent flow:

A Tollmien–Schlichting wave may be set up in the boundary layer by the selective amplification of pre-existing small disturbances. As the wave travels downstream, it is amplified or damped in the way predicted by the linearized stability theory. At the same time the wave is deformed into a three-dimensional configuratoin by the pre-existing three-dimensionality such as residual spanwise variation of mean flow. So long as the wave remains small in intensity, however, it is damped out eventually and does not result in the onset of turbulence. It is only after the wave intensity exceeds a certain amount which depends on the degree of pre-existing three-dimensionality, that the nonlinear development takes place which is characterized by the rapid amplification of wave intensity, the rapid increase in wave three-dimensionality, and the distortion in mean velocity profile. The appearance of nonlinear development inevitably leads to the breakdown of laminar flow, and hence the onset of turbulence. There is present a mechanism by which the energy is transferred from one spanwise position to another so that the breakdown of laminar flow occurs as a consequence of three-dimensional development of the wave as a whole.

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